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A three-level supply chain with warranty services, pricing and marketing decisions

Competition and cooperation analysis

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Abstract

Purpose – This paper aims to present various three-level service contracts among the following three participants: a manufacturer, an agent and a customer. The interaction between the aforementioned participants will be modeled using the game theory approach. Under non-cooperative and cooperative games, the optimal sale price, warranty period and warranty price for the manufacturer and the optimal maintenance cost (repair cost) and marketing expenditure for the agent are obtained by maximizing their profits. The satisfaction of the customer is also maximized by being able to choose one of the suggested options from the manufacturer and the agent, based on the risk parameter.

Design/methodology/approach – Three-echelon supply chains with marketing and warranty services are studied. Game-theoretic approaches (non-cooperative and cooperative) are presented. The non-cooperative approaches are static (NE) and dynamic (Stakelberg) models. The cooperative approach is related to bargaining models (Nash bargaining games). The authors develop a sensitivity analysis of some parameters and their effect.

Findings – Based on the mentioned drawbacks (i.e. lack of a model containing warranty, marketing and pricing), despite their importance, a developed model is proposed in this research to cover one of the research gaps. In addition, main contributions of this paper that differentiate it from the existing papers are regarding inventory, lost sale and lost goodwill, which are significant in the comparison environment. Another advantage of this study is related to the solution approach, the game theory. Twofold of the games theoretical, i.e. cooperative (in three forms) and non-cooperative are considered, because of their importance. Three types of non-cooperative games are presented as follows: Nash equilibrium – each echelon decides respectively and simultaneously; manufacturer-Stackelberg – the manufacturer has more power than the agent and the agent has more power than the customer; and customer-Stackelberg – customer is leader of the agent and the agent is the leader of manufacturer. The involved cooperative game in this paper is the bargaining problem that the participants can determine how to share the additional profits.

Originality/value – In this paper, various three-level service contracts will be presented among the following three participants: a manufacturer, an agent and a customer. The interaction between the aforementioned participants will be modeled using the game theory approach. Under non-cooperative and cooperative games, the optimal sale price, warranty period and warranty price for the manufacturer and the optimal maintenance cost (repair cost) and marketing expenditure for the agent are obtained by maximizing their profits. The satisfaction of the customer is also maximized by being able to choose one of the suggested options from the manufacturer and the agent, based on the risk parameter. Several numerical examples are used to illustrate the models presented in this paper. Finally, the authors develop a sensitivity analysis of some parameters and their effects on the objective functions.

Keywords Marketing, Game theory, Warranty service, Three-level supply chain

Paper type Research paper

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1. Introduction

Supply chain management usually includes various echelons, such as manufacturer, retailer and customer, where each one has its own rights and individual interests (Palsule-Desai *et al.*, 2013). In these circumstances, the managers often consider some contrasts between themselves to coordinate, improve the overall effectiveness of the whole supply chain and maximize the total profit; the managers often consider some contracts between echelons. Recently, the contract concepts have received great attention in both industry and academia. One of the service contrasts is the warranty that it is a key strategic tool to obtain more sales, revenue and profit (Kurata and Nam, 2013). Warranty is determined contractually by principles of sales and it encourages customers to purchase more products (Wei *et al.*, 2014). During the past two decades, an increase in the application of warranties as competitive tools has been seen, especially in durable consumer product market (DeCroix, 1999).

In addition to the warranty service, marketing and advertising play a significant role in conventional supply as an instrument to increase sales. It is widely used in practice and as a strategy to improve the performance of supply chain network (Jørgensen and Zaccour, 2014; Krishnamoorthy *et al.*, 2010; Xie and Neyret, 2009; Li *et al.*, 2002; and Yang *et al.*, 2012). One of the contrasts is pricing which is crucial and determines company's profit and survival (Hua *et al.*, 2010). Manufacturers and retailers lure customers and boost sales by pricing strategies (Wei *et al.*, 2014). The most current approach to handle the mentioned concepts (i.e. marketing, warranty and pricing) is game theoretical model (SeyedEsfahani *et al.*, 2011), and there is a rich literature in this area over the past three decades (Ingene *et al.*, 2012) so that two review papers were published about this field (Jørgensen and Zaccour, 2014; Krishnamoorthy *et al.*, 2010; Xie and Neyret, 2009; Li *et al.*, 2002; Yang *et al.*, 2012; Hua *et al.*, 2010; SeyedEsfahani *et al.*, 2011; Ingene *et al.*, 2012; Aust and Buscher, 2014). Based on the mentioned texts, a literature review is surveyed, and some the related researches are summarized in Table I.

Li *et al.* (2002) developed three models to determine equilibrium marketing and investment for a two-echelon supply chain, and they considered one cooperative bargaining model to select the best cooperative advertising expenditure-sharing rule between the manufacturer and the retailer and also the Stackelberg games.

Parlar and Weng (2006) modeled a problem under two scenarios – no coordination and coordination in both firms – and used the Nash strategy to analyze two scenarios. It was shown that coordinating the pricing and production quantity decisions may result in increasing expected profits. Jackson and Pascual (2008) presented a Nash game to bargain pricing in service contracts share expected profits. In that paper, optimal maintenance service contract is determined by considering the realistic case.

Karray and Martín-Herrán (2009) analyzed relationship between the manufacturer's and the retailer's advertising and pricing strategies. The authors considered a differential game that incorporates the effects of advertising. It is concluded that relationship between advertising and pricing decisions depends on the nature of the advertising effects and also the retailer's advertising, as a strategic decision should be regarded by both of the manufacturer and retailer. Four various models via pricing and cooperative advertising strategies are studied based on three non-cooperative games and one cooperative game by Xie and Neyret (2009). The non-cooperative games contain Nash, Stackelberg retailer and Stackelberg manufacturer, and the considered cooperative game is Nash bargaining. The results present a clear picture about the competition and cooperation via pricing and cooperative advertising strategies for the participants.

Xie and Wei (2009) developed two models which the demand is determined by retail price and cooperative advertising by members. The optimal equilibrium pricing and cooperative

Table I.
Literature review

Articles	Free replacement	Warranty Outsourcing services	Maintenance options	Marketing Cooperative	Non-cooperative	Pricing Dynamic	Static
Li *et al.* (2002)				✓			
Parlar and Weng (2006)				✓	✓		✓
Jackson and Pascual (2008)		✓					
Karray and Martín-Herrán (2009)				✓		✓	
Xie and Neyret (2009)				✓			✓
Xie and Wei (2009)				✓			✓
Esmaeili *et al.* (2009)					✓		✓
Kurata and Nam (2010)				✓			
Krishnamoorthy *et al.* (2010)					✓	✓	
Hua *et al.* (2010)					✓		✓
SeyedEsfahani *et al.* (2011)				✓			✓
Yan (2011)				✓			
Kunter (2012)				✓			
Ingene *et al.* (2012)					✓		✓
Chen *et al.* (2012)	✓		✓				✓
Wu (2012)			✓				✓
Zhao *et al.* (2012)							✓
Kurata and Nam (2013)				✓			
Ma *et al.* (2013)				✓			
Yang *et al.* (2012)				✓			
Parlar and Weng (2006)					✓		
Lu and Liu (2013)					✓		✓
Esmaeili *et al.* (2014)	✓	✓	✓				✓
Wei *et al.* (2014)			✓				✓
Wu (2014)			✓				
Avinadav *et al.* (2014)							✓
Hsieh *et al.* (2014)							✓
Zhao and Wang (2015)					✓		✓
Sana *et al.* (2018)				✓	✓		
Taleizadeh *et al.* (2018)							✓
Saha *et al.* (2018)							✓
Modak *et al.* (2018)							✓
Current research	✓	✓	✓		✓		✓

(*continued*)

Articles	Demand function				Game theory			
					Cooperative		Non-cooperative	
	Linear	Exponential	Quadratic	Radically	Bargaining	Others	Nash	Stackelberg
Li *et al.* (2002)		✓			✓			✓
Parlar and Weng (2006)	✓						✓	
Jackson and Pascual (2008)					✓		✓	
Karray and Martín-Herrán (2009)	✓						✓	✓
Xie and Neyret (2009)	✓				✓		✓	✓
Xie and Wei (2009)	✓				✓			✓
Esmaeili *et al.* (2009)		✓					✓	✓
Kurata and Nam (2010)			✓				✓	
Krishnamoorthy *et al.* (2010)	✓						✓	
Hua *et al.* (2010)	✓							✓
SeyedEsfahani *et al.* (2011)		✓		✓		✓	✓	✓
Yan (2011)	✓				✓			✓
Kunter (2012)				✓	✓			✓
Ingene *et al.* (2012)	✓						✓	✓
Chen *et al.* (2012)	✓						✓	✓
Wu (2012)	✓						✓	
Zhao *et al.* (2012)	✓				✓			✓
Kurata and Nam (2013)			✓				✓	
Ma *et al.* (2013)	✓				✓			✓
Yang *et al.* (2012)	✓							✓
Parlar and Weng (2006)	✓						✓	✓
Lu and Liu (2013)	✓						✓	✓
Esmaeili *et al.* (2014)		✓					✓	✓
Wei *et al.* (2014)	✓				✓		✓	✓
Wu (2014)	✓							✓
Avinadav *et al.* (2014)				✓				✓
Hsieh *et al.* (2014)	✓						✓	✓
Zhao and Wang (2015)	✓						✓	✓
Sana *et al.* (2018)						✓		✓
Taleizadeh *et al.* (2018)	✓							✓
Saha *et al.* (2018)	✓					✓		✓
Modak *et al.* (2018)	✓				✓			✓
Current research		✓			✓		✓	✓

Table I.

advertising strategies are identified in channel coordination between a manufacturer using two game-theoretic models (cooperative and non-cooperative games). Five analytical models are presented by Kurata and Nam (2010) to warranty service. Nash and Stackelberg games are applied to solve the models. It is presented that warranty service do not guarantee the optimal that can satisfy customers.

Krishnamoorthy *et al.* (2010) proposed a model of marketing and price competition in a dynamic environment. The optimal advertising and pricing decisions for both symmetric and asymmetric competitors are obtained using differential game theory. Hua *et al.* (2010) surveyed the decisions for the delivery lead time and prices in a supply chain using the two-stage optimization technique and Stackelberg game. It is shown that delivery lead time impact the manufacturer's and retailer's pricing strategies and profits. SeyedEsfahani *et al.* (2011) suggested four game-theoretic models to the vertical cooperative advertising along with pricing strategies in a two-echelon supply chain. The four scenarios included Nash game, Stackelberg manufacturer game, Stackelberg retailer game and a cooperative game.

Yan (2011) considered a cooperation channel through differentiated branding and profit-sharing decisions as a mathematical model. Both members, the manufacturer and the retailer, can obtain the full coordination and also their individual profits are increased in the cooperation state. Kunter (2012) designed a contract to establish efficiency in a manufacturer–retailer channel coordination. Manufacturer and retailer select an efficient contract by bargaining over the wholesale price. Ingene *et al.* (2012) integrated the game-theoretic literature on coordination of distribution channels without competition and developed a general model that covers the most main models in the marketing literature. Chen *et al.* (2012) considered a supply chain containing a manufacturer and two competing retailers, with warranty-time-dependent demands. With the application of the game theory, the optimal warranty time, optimal wholesale price and the optimal profit of the members under the various pricing strategies are obtained in the research. Wu (2012) incorporated service competition between two manufacturers that bundle the products with services, containing warranty and advertisement. The first manufacturer produces a new product and the second produces a remanufactured product. According to a theoretical and numerical analyses, economic and managerial insights for the retailer and the manufacturer are obtained.

Zhao *et al.* (2012) presented four different expected value models in an uncertain environment that demands and manufacturing costs are handled as the fuzzy number. The pricing strategies (wholesale prices and retail prices) are identified using game theory approach. The impact of uncertainty on warranty service strategies is examined in a two-stage supply chain by Kurata and Nam (2013). It is presented that uncertainty reduces the variation of the warranty service between the firm's decisions and the customers' optimal service levels according to maximizing profits under the Nash game framework. Ma *et al.* (2013) considered three different contracts under the decentralized model and proposed an effective supply chain contract based on two kinds of conventional contracts: two-part tariff contract and cost-sharing contract. They obtain the optimal levels of retail sales effort, quality-improvement effort and optimal total profit. Yang *et al.* (2012) proposed a game theory model including one manufacturer and one retailer. Advertising is used by retailer to improve the sales, and a proportion of the advertising cost is shared by the manufacturer. The best choice of the manufacturer is obtained as the Stackelberg game.

Palsule-Desai *et al.* (2013) developed a non-cooperative game theory model to analyze stability concepts in a supply chain with a marketing agent coordinating activities using a price and profit-sharing-based coordination mechanism in a real-life network. Ranges for cost, number of players and the profit-sharing parameter over which the network is stable are obtained. Lu and Liu (2013) studied the impact of pricing, game plans and efficiency of e-channels on the selling prices, wholesale prices and profits of the supplier and retailer. Three types of pricing games (including two kinds of Stackelberg and a Nash) are investigated. It is shown that the efficiency of an e-channel impact on supplier and retailer profits remarkably. The price and warranty period strategies in a supply chain including two manufacturers and one retailer under decentralized decision mode are considered by Wei *et al.* (2014) from a two-stage game theoretic perspective. It is shown, when the two manufacturers adopt the cooperation strategy, the members gain more profits, as well as lower retail prices and longer warranty periods. Wu (2014) used the renewal-reward process to estimate the number of warranty claims for the first time. The authors presented three warranty return strategies about sending the new items to warranty claimants or not. The optimal warranty periods are determined using game theory. Avinadav *et al.* (2014) consider a supply chain containing a manufacturer and a retailer with the uncertain demand which it depends on price and monetary investment. In that paper, three supply chain models are studied, including manufacturer Stackelberg, retailer Stackelberg and vertical integration. It is shown that the manufacturer can improve its profit, if its leadership to the retailer is left and or the power balance is in the determined desirable limits. Hsieh *et al.* (2014) developed a decentralized uncoordinated model and a centralized integrated model related to policies of prices and stocking quantities. The supply chain includes multiple manufacturers and a retailer with the uncertain demand. It is exhibited that the decentralized system will attain the centralized optimal profit and each participants obtains the more profit than with the decentralized uncoordinated system if the profit assignment conditions are met. The pricing and retail service decisions of a two-echelon supply chain are examined by Zhao and Wang (2015). The customer demands, manufacturing costs and service cost coefficients are assumed as fuzzy numbers. The corresponding analytical equilibrium solutions are obtained using three games: manufacturer-leader Stackelberg, retailer-leader Stackelberg and vertical nas. Esmaeili *et al.* (2014) pointed to several types of warranty and presented several models containing warranty strategies. Esmaeili *et al.* (2009) proposed the seller–buyer supply chain models considering marketing and pricing.

Sana *et al.* (2018) proposed a methodology to analyze and enhance the financial relationship between a manufacturer and a distributer. A real case was studied to validate the proposed model as well, and the output results illustrated that invest in technology is necessary to make the strategic improvement decisions. Taleizadeh *et al.* (2018) presented a mathematical model for airport pricing with congested airports. A four-game theoretic strategy is suggested to select the best decision for air traveling companies. Some sensitivity analysis and numerical examples showed the efficiency of the proposed model, and finally it was concluded that Stackelberg–Cournot behavior is the most effective strategy. A two-echelon supply chain comprising manufacturer and retailer for cooperative and non-cooperative scenarios under inconsistent and consistent pricing was proposed and analyzed by Saha *et al.* (2018). In that paper, price of the product and delivery lead time are considered as crucial factors in customers' purchase decisions and demand is sensitive to them.

Sana *et al.* (2017) suggested a bi-level optimization model to maximize the profit of the channel members in an agro-industry chain of cocoa. Two collaborative and non-collaborative systems are compared, and it is concluded that the collaborative is more appropriate, which depends on the theory and practice of the supply chain management. Modak *et al.* (2018) addressed the best channel structure of closed-loop supply chain containing some decisions, namely, price, product quality and recycling rate. A model is formulated to use alternating offer bargaining for channel coordination. Also, product recycling, quality level of the product and coordination of the two-echelon closed-loop supply chain are merged.

Literature review indicates that there is a need to propose a model considering the critical and effective issues, namely, warranty, marketing and pricing. To the best of the authors' knowledge, a research investigating the above-mentioned issues simultaneously has not been addressed in the literature yet. As it can be derived from Table I, there is a limit studies considering the warranty contract which is a powerful tool to succeed in a competitive world (Wei *et al.*, 2014 and DeCroix, 1999). In addition, main contributions of this paper that differentiate this paper from the existing ones are regarding inventory, lost sale and lost goodwill, which are crucial in the comparison environment. Considering the risk parameter brings the proposed model closer to real world, the proposed model is more general, in line with the real world, and it overcomes many of the presented research gaps and the suggestions of the previous researchers. Moreover, another advantage of this study is related to the solution approach, the game theory. Twofold of the games theoretical, i.e. cooperative (in three forms) and non-cooperative, is considered because of their importance. Three types of non-cooperative games are presented as follows:

(1) *Nash equilibrium*: Each echelon decides respectively and simultaneously.
(2) *Manufacturer-Stackelberg*: The manufacturer has more power than the agent and the agent has more power than the customer.
(3) *Customer-Stackelberg*: The customer is the leader of the agent and the agent is the leader of manufacturer.

The involved cooperative game in this paper is the bargaining problem that the participants can determine how to share the additional profits. As it can be seen from Table I, there are only a few papers considering the various types of game theory approaches.

The remainder of the current study is structured as follows. The problem is described in Section 2, and Section 3 is about the formulation of the developed models separately. Then non-cooperative games and a cooperative approach are presented in Section 4. In Section 5, a few numerical examples are discussed to verify the proposed approach and the model. Section 6 studies the sensitivity analysis of the some parameters and their effects on the objective functions value. Finally, the concluding remarks and directions of the future study are expressed in Section 7.

2. Models

In this paper, we rely on the models presented by Esmaeili *et al.* (2014) to which we add the notions of marketing, inventory, shortages and goodwill, notable in both industry and academia. Consider a manufacturer who produces and sells the product. He can do it with or without warranty. If the customer chooses warranty, the manufacturer has to replace the failed product by a new one during the warranty period. There is a third party, an agent, whose role in the chain is to repair the failed product without warranty and/or the products whose warranty time is expired. The agent may offer two options to the customer:

(1) pay a maintenance cost and all failures are repaired for free; or
(2) repair the failed product at a fixed cost per failure.

Therefore, the customer faces three items:

(1) with warranty;
(2) without warranty but with maintenance cost; or
(3) without warranty but with repair cost.

In short, all of the models will be described.

First, we describe the assumptions involved in the models:

- Planning horizon is infinite.
- Parameters are deterministic and known in advance.
- There are one customer, one agent and one manufacturer.
- The failure intensity is given by λ = the initial failure rate + the age of product × rate of aging.
- The number of product failures follows a Poisson distribution with intensity λ.
- The ordering, waiting and repairing time are disregarded.
- Demand follows an exponential distribution function.
- Shortage cost is considered as lost sale and goodwill costs

In addition, all decision variables and input parameters that will be used to formulate the models are stated.

Decision variables:

p_1 = sale price given by the manufacturer when no warranty is offered;
p_2 = sale price given by the manufacturer when warranty is offered;
t = warranty time;
n = warranty price paid by the customer to the manufacturer;
r_1 = repair cost charged by the agent to the customer for a fixed cost per failure after the warranty extended by the manufacturer expires;
u = maintenance price announced by the agent to the customer for all failures;
r_3 = repair cost charged by the agent to the customer for per failure;
m_1 = marketing expenditure incurred by the agent when warranty is offered;
m_2 = marketing expenditure incurred by the agent when all failures are repaired for a fixed maintenance price; and
m_3 = marketing expenditure incurred by the agent when the failed product are repaired at a fixed cost per failure.

Parameters:

c = Production cost of the product in the manufacturer;
s = Salvage value of the failed product;
h = Inventory holding cost of the product in the manufacturer;
μ = Rate of lost sale;
b_1 = Lost sale cost;
b_2 = Lost goodwill cost;
g = Amplitude constant factor;
λ = Failure rate;
w = Rate of ageing;
l = lifetime of product;
δ = Scaling constant for warranty price;
v = Repair cost incurred by the agent;
d' = Product volume sold by the manufacturer when warranty is offered;
d = Product volume sold by the manufacturer when no warranty is offered;
d_1 = Product volume proceed by the agent when warranty is offered;

d_2 = Product volume proceed by the agent when warranty is offered and all failures are repaired for a fixed maintenance price;
d_3 = Product volume proceed by the agent when warranty is offered and the failed product are repaired at a fixed cost per failure;
α_1 = Price elasticity of the manufacturer when warranty is offered;
α_2 = Price elasticity of the manufacturer when no warranty is offered;
β_1 = Price elasticity of the agent when warranty is offered and the warranty is expires;
β_2 = Price elasticity of the agent when all failures are repaired for a fixed maintenance price;
β_3 = Price elasticity of the agent when the failed product are repaired at a fixed cost per failure;
θ_1 = Marketing expenditure elasticity of demand when no warranty is offered, ($\theta_1 < \beta_1$);
θ_2 = Marketing expenditure elasticity of demand when all failures are repaired for a fixed maintenance price, ($\theta_2 < \beta_2$); and
θ_3 = Marketing expenditure elasticity of demand when the failed product are repaired at a fixed cost per failure, ($\theta_3 < \beta_3$).

The proposed models for the manufacturer, agent and customer will be mathematically formulated below.

2.1 Manufacturer's models

As already mentioned, the manufacturer offers the following two options to the customer:

(1) *M1*: The failed item will be replaced during the warranty period free of charge (with warranty).
(2) *M2*: No warranty will be given (without warranty).

Based on the two options, the optimal solutions of the manufacturer's profit functions are obtained.

2.1.1 With warranty. The related profit function here is:

$$\begin{aligned}&\prod\nolimits_{M1}(p_1, t) = d'\left[p_1 + n - c - (c-s)\left(\lambda t + 0.5wt^2\right) - h - \mu(b_1 + b_2)\right]\\&s.t.: d' = gp_1{}^{-\alpha_1} r_1{}^{-\beta_1} t^{-\alpha_1} n^{-\alpha_1}\\&\qquad n = \delta t\end{aligned} \tag{1}$$

This model has an advantage over the one proposed by Esmaeili *et al.* (2014), sales volume depends on the warranty period and warranty price that it is disregarded beforehand. As the profit function is a concave function in p_1 and t (refer to Appendix *AP1*), the optimal time warranty and the sale price for the manufacturer are obtained by applying the first-order deviation to (1) resulting in:

$$p_1^* = \frac{\alpha_1(1-2\delta) - \lambda(c-s)[2 + \alpha_1\lambda(c+s)] + 2\alpha_1\left[(c-s)(hw + \delta\lambda + w\mu(b_1+b_2)) + c\left(\mathrm{w}(\mathrm{c}-\mathrm{s}) - s\lambda^2\right)\right] + 2}{2\alpha_1 w(c-s)} \tag{2}$$

$$t^* = \frac{\alpha_1[\lambda(c-s) - 2\delta + 1] + 2}{\alpha_1 w(c-s)}. \tag{3}$$

2.1.2 Without warranty. For the second option, M_2, we should consider two possibilities of the agent: (A_2) the customer pays a maintenance price to the agent and all failures are repaired free of charge and (A_3) the customer pays a repair cost per failure to the agent.

Now the profit function for the manufacturer is:

$$\prod_{M2}(p_2) = d''(p_2 - c - h - \mu(b_1 + b_2))$$
$$s.t.{:}d'' = g p_2^{-\alpha_2}(q_1 u + q_2 r_3)^{-\beta_2 q_1 - \beta_3 q_2} \tag{4}$$
$$q_1 + q_2 = 1$$

q_1 and q_2 are binary variables, based on the customer selection from the agent options.

As the profit function is a concave function in p_2 (refer to Appendix *AP2*), the optimal sale price is obtained by applying the first-order condition to (4) resulting in:

$$p_2^* = \frac{\alpha_2[c + h + \mu(b_1 + b_2)]}{\alpha_2 - 1}. \tag{5}$$

2.2 Agent's models

The agent offers three options to the customer:

(1) *A1*: Warranty is considered and the customer will pay a warranty price. After warranty times the customers refer to the agent to repair the failed product (with warranty).

(2) *A2*: The customer pays a maintenance price to the agent and all failures are repaired free of charge (without warranty but with maintenance cost).

(3) *A3*: The customer pays a repair cost per failure to the agent (without warranty but with repair cost).

Based on the three options of the agent the optimal solutions of profit functions are obtained as follows.

2.2.1 With warranty. The profit function for the agent is:

$$\Pi_{A1}(r_1, m_1) = d_1\left[(r_1 - v)\left(\lambda(l - t) + 0.5w(l - t)^2\right) - m_1\right]$$
$$s.t.{:}\ \ d_1 = g p_1^{-\alpha_1} r_1^{-\beta_1} m_1^{\theta_1} \tag{6}$$

The profit function is a strictly pseudoconcave function in r_1 for fixed m_1 (refer to Appendix *AP3*). Therefore, the optimal repair cost and marketing expenditure are:

$$m_1^* = \frac{\theta_1 v\left(\lambda(l - t) + 0.5w(l - t)^2\right)}{\beta_1 - \theta_1 - 1} \tag{7}$$

$$r_1^* = \frac{\beta_1 v}{\beta_1 - \theta_1 - 1} \tag{8}$$

2.2.2 Without warranty. For the option without warranty, we should consider two options for the customer: (A_2) paying a maintenance price to the agent and all failures are repaired free of charge (without warranty but with maintenance cost); and (A_3) paying a repair cost per failure to the agent (without warranty but with repair cost).

2.2.2.1 With maintenance cost. In this first option, the products are sold without warranty, the customers pay a maintenance cost and when the products fail, they refer to the agent to repair the failed product for free.

Now the profit function here is:

$$\Pi_{A2}(u, m_2) = d_2\left[u - m_2 - v(\lambda l + 0.5wl^2)\right]$$
$$s.t.\text{: } d_2 = gp_2^{-\alpha_2}u^{-\beta_2}m_2^{\theta_2} \tag{9}$$

The profit function is concave in u and m_2 (refer to Appendix *AP4*). Then, the optimal marketing expenditure and maintenance price are:

$$m_2^* = \frac{\theta_2 v(\lambda l + 0.5wl^2)}{\alpha_2 - \theta_2 - 1} \tag{10}$$

$$u^* = \frac{\alpha_2 v(\lambda l + 0.5wl^2)}{\alpha_2 - \theta_2 - 1}. \tag{11}$$

2.2.2.2 With repair cost. For the second option without warranty, the products can be failed. If so, the customers pay a repair cost per failure to the agent. The profit function is:

$$\Pi_{A3}(r_3, m_3) = d_3\left[(r_3 - v)(\lambda l + 0.5wl^2) - m_3\right]$$
$$s.t.\text{: } d_3 = gp_2{}^{-\alpha_2}r_3{}^{-\beta_3}m_3^{\theta_3} \tag{12}$$

The profit function is concave in r_3 and m_3 (refer to Appendix *AP5*). Therefore, the optimal marketing expenditure and repair cost are:

$$m_3^* = \frac{\theta_3 v(\lambda l + 0.5wl^2)}{\beta_3 - \theta_3 - 1} \tag{13}$$

$$r_3^* = \frac{\beta_3 v}{\beta_3 - \theta_3 - 1} \tag{14}$$

2.3 Customer's models

In the costumer models, the risk parameter on the customer side is considered. Because the agent would offer only the last two options if the manufacturer provided no warranty, the customer would have three options to choose from:

(1) *C1 (M1 and A1)*: Pay p_2 to the manufacturer for price of warranty and after its expiration, pay a repair cost of r_1 per failure to the agent (with warranty).

(2) *C2 (M2 and A2)*: Pay a maintenance price u to the agent; thereafter, each failure would be repaired free of charge (without warranty but with maintenance cost).

(3) *C3 (M2 and A3)*: Pay a repair cost of r_3 per failure to the agent during the lifetime of the product (without warranty but with repair cost).

Based on the three options of the customer the optimal solutions of profit functions are obtained.

2.3.1 With warranty. Similar to Esmaeili *et al.* (2014), under option C_1, the customer's quantitative satisfaction function and the customer's utility function considering the risk effect are, respectively, as follows:

$$\prod_{c1} = d_1\left[\rho l - p_1 - n - r_1\left(\lambda(l-t) + 0.5w(l-t)^2\right)\right] \tag{15}$$

$$U_{c1} = \left[1 - \exp\left(-\gamma_1 d_1\left(\rho l - pt - p_1 - n - r_1\left(\lambda(l-t) + 0.5w(l-t)^2\right)\right)\right]/\gamma_1 \tag{16}$$

2.3.2 Without warranty. For the option without warranty, we should consider two options for the customer: (C_2) paying a maintenance price to the agent and all failures are repaired free of charge (without warranty but with maintenance cost); and (C_3) paying a repair cost per failure to the agent (without warranty but with repair cost).

2.3.2.1 With maintenance cost. When the customer chooses option C_2, the quantitative satisfaction and the utility functions would be given as:

$$\Pi_{c2} = d_2(\rho l - p_2 - u) \tag{17}$$

$$U_{c2} = [1 - \exp(-\gamma_2 d_2(\rho l - p_2 - u))]/\gamma_2 \tag{18}$$

2.3.2.2 With repair cost. Finally, when the customer chooses option C_3, the satisfaction and the utility are:

$$\Pi_{c3} = d_3[\rho l - p_2 - r_3(\lambda l + 0.5wl^2)] \tag{19}$$

$$U_{c3} = \left[1 - \exp\left(-\gamma_3 d_3(\rho l - p_2 - r_3 - (\lambda l + 0.5wl^2))\right)\right]/\gamma_3 \tag{20}$$

3. Supply chain games

As we have already stated, the interaction between the manufacturer, agent and customer is modeled using a game theory approach, and this covers both non-cooperative and cooperative games. In the non-cooperative game, two types of scenarios are considered. In the first scenario, Nash equilibrium is obtained while the manufacturer, agent and customer choose their strategies separately and simultaneously. In the second scenario, we assume there exists a certain asymmetry of power between the players, which generates two dynamic models. In the first model, the manufacturer has more power than the agent (Manufacturer Stackelberg game) and the agent has more power than the customer (Agent Stackelberg game). As for the second, the customer has more power than the agent (Customer Stackelberg game) and the agent has more power than the manufacturer (Agent Stackelberg game). In both models, sub-game perfect equilibrium (SPE) can be obtained by the backward induction method. In the cooperative game, we consider a bargaining problem in which all the players (manufacturer, agent and customer) cooperate together and act as an integrated service–provider–customer to reach an agreement that allows them to get a few extra benefits. In the following, we present the non-cooperative games (static and Stackelberg models) and the bargaining game.

3.1 Static game

We consider a three-person non-cooperative game and for each player, we also specify the set of strategies and the corresponding profit available to that player.

- $N = \{M, A, C\}$ where the manufacturer, the agent and the customer are represented by M, A and C, respectively.
- The set of strategies available to the player $i \in \{M, A, C\}$ is:

$$S_M = \{M_1, M_2\},$$

$$S_A = \{A_1, A_2, A_3\}$$

$$S_C = \{C_1, C_2, C_3\}.$$

- The payoff $\Pi(i)$ of each player $i \in \{M, A, C\}$ is:

$$\Pi(M) = \{\Pi_{M1}(p_1, t), \Pi_{M2}(p_2)\}$$

$$\Pi(A) = \{\Pi_{A1}(r_1, m_1), \Pi_{A2}(u, m_2), \Pi_{A3}(r_3, m_3)\}$$

$$\Pi(C) = \{U_{C1}, U_{C2}, U_{C3}\}.$$

The players obtain their best strategy (one that maximizes their own profit/utility) simultaneously and separately. Based on the previous description, our three-person game is a finite game (finite number of players and strategies); thus, we are sure that our static game has at least one Nash equilibrium (Nash, 1951). Nash equilibrium provides an optimal solution or a best response of the manufacturer, agent and customer's model.

Table II allows us to select the Nash equilibrium, i.e. the best response of the manufacturer, agent and customer's models. The profit of the manufacturer is obtained using optimal sale price and warranty time when warranty is offered or not, regardless of what the agent and customer are doing. The profit of agent is obtained by optimal repair, maintenance and marketing costs. Moreover, the utility function of the customer according to the different options.

3.2 Dynamic games

In the dynamic non-cooperative game, we consider a certain asymmetry of power between the players. Stackelberg strategy is used when such an asymmetry power is assumed between the players or in their turns. The decision-making is not simultaneously but

Table II.
Three-player normal form game

		C_1	C_2	C_3
A_1	M_1	$(\Pi_{M1}, \Pi_{A1}, U_{C1})$	$(\Pi_{M1}, \Pi_{A1}, U_{C2})$	$(\Pi_{M1}, \Pi_{A1}, U_{C3})$
	M_2	$(\Pi_{M2}, \Pi_{A1}, U_{C1})$	$(\Pi_{M2}, \Pi_{A1}, U_{C2})$	$(\Pi_{M2}, \Pi_{A1}, U_{C3})$
A_2	M_1	$(\Pi_{M1}, \Pi_{A2}, U_{C1})$	$(\Pi_{M1}, \Pi_{A2}, U_{C2})$	$(\Pi_{M1}, \Pi_{A2}, U_{C3})$
	M_2	$(\Pi_{M2}, \Pi_{A2}, U_{C1})$	$(\Pi_{M2}, \Pi_{A2}, U_{C2})$	$(\Pi_{M2}, \Pi_{A2}, U_{C3})$
A_3	M_1	$(\Pi_{M1}, \Pi_{A3}, U_{C1})$	$(\Pi_{M1}, \Pi_{A3}, U_{C2})$	$(\Pi_{M1}, \Pi_{A3}, U_{C3})$
	M_2	$(\Pi_{M2}, \Pi_{A3}, U_{C1})$	$(\Pi_{M2}, \Pi_{A3}, U_{C2})$	$(\Pi_{M2}, \Pi_{A3}, U_{C3})$

sequentially. The most powerful player or first starter is named leader and someone how select the best response based on the leader's decision is identified as a follower.

Two cases are presented here:

1. (1) The manufacturer has more power than the agent and the agent has more power than the customer (Manufacturer Stackelberg game).
2. (2) The customer has more power than the agent and the agent has more power than the manufacturer (Customer Stackelberg game).

The main reason for considering those models is the relationship between the large companies, which can be seen today. As Esmaeili *et al.* (2014) points out, BMW4 offers third-party warranty in which the manufacturer dominates the agents on the price of warranty and the type of service contract. To obtain SPE, the backward induction method can be used because both models are finite games with perfect recall (Fudenberg and Tirole, 1991). Therefore, the leader makes the first move and the follower then reacts by playing the best move consistent with the available information.

3.2.1 Manufacturer-Stackelberg. As Esmaeili *et al.* (2014), we assume that the manufacturer has more power than the agent and the agent has more power than the customer. Therefore, the manufacturer selects the best strategy by dominating the agent and the agent, by dominating the customer in a conventional way.

The profit function of the agent is maximized according to the utility functions of the customer as follows:

$$\begin{aligned}\max \Pi_A(r_1,u,r_3,m_1,m_2,m_3) &= z_1d_1\left[(r_1-v)\left(\lambda(l-t)+0.5w(l-t)^2\right)-m_1\right]\\ &+z_2d_2\left[u-v(\lambda l+0.5wl^2)-m_2\right]+z_3d_3\left[(r_3-v)(\lambda l+0.5wl^2)-m_3\right]\\ s.t.:&\\ z_1U_{c1} &= \left[1-\exp\left(-\gamma_1d_1\left(\rho l+\rho t-p_1-n-r_1\left(\lambda(l-t)+0.5w(l-t)^2\right)\right)\right)\right]/\gamma_1\\ z_2U_{c2} &= \left[1-\exp(-\gamma_2d_2(\rho l+p_2-u))\right]/\gamma_2\\ z_3U_{c3} &= \left[1-\exp\left(-\gamma_3d_3\left(\rho l+p_2-r_3(\lambda l+0.5wl^2)\right)\right)\right]/\gamma_3\\ z_1+z_2+z_3 &= 1\end{aligned} \quad (21)$$

z_i are binary variables. If $z_1 = 1$, the first option of the agent is selected and optimal r_1 and m_1 are given by:

$$r_1^* = \frac{\ln(1-\gamma_1z_1U_1)+\gamma_1d_1(\rho l+\rho t-p_1)}{\gamma_1d_1\left[\left(\lambda(l-t)+0.5w(l-t)^2\right)\right]} \quad (22)$$

$$m_1^* = \frac{\theta_1\ln(1-\gamma_1z_1U_1)+\gamma_1d_1(\rho l+\rho t-p_1)}{\gamma_1d_1(\beta_1-\theta_1-1)\left[\lambda(l-t)+0.5w(l-t)^2\right]} \quad (23)$$

If $z_2 = 1$, the second option of the agent is the best decision and optimal u and m_2 are obtained as follows:

$$u^{*} = \rho l - p_2 + \ln(1 - \gamma_2 d_2 u_2)/\gamma_2 d_2 \tag{24}$$

$$m_2^{*} = \frac{\theta_2}{\theta_2 + 1}\left[\rho l - p_2 - v(\lambda l + 0.5wl^2) + \ln(1 - \gamma_2 d_2 u_2)/\gamma_2 d_2\right] \tag{25}$$

If $z_2 = 1$, the third option of the agent is chosen and optimal r_3 and m_3 are obtained as follows:

$$r_3^{*} = \left[\rho l - p_2 + \ln(1 - \gamma_3 z_3 U_3)/\gamma_3 d_3\right]/\left[\lambda l + 0.5wl^2\right] \tag{26}$$

$$m_3^{*} = \frac{\theta_3}{\theta_3 + 1}(\lambda l + 0.5wl^2)\left[\rho l - p_2 - v + \ln(1 - \gamma_3 z_3 U_3)/\gamma_3 d_3\right] \tag{27}$$

Because the manufacturer is the leader of agent, these optimal values should be replaced in the manufacture's profit function to get the best response by determining price sale, warranty price and warranty time. Thus the manufacturer's profit function is formulated as follows:

$$\max \Pi_M(p_1, p_2, n, t) = y_1 d'\left[p_1 + n - c(c - s)(\lambda t + 0.5wt^2) - h - \mu(b_1 + b_2)\right] + y_2 d''(p_2 - c - h - \mu(b_1 + b_2)) \tag{28}$$

$$\begin{aligned}
&s.t.:\\
&r_1^{*} = y_1 \frac{\ln(1 - \gamma_1 z_1 U_1) + \gamma_1 d_1(\rho l + \rho t - p_1)}{\gamma_1 d_1\left[\left(\lambda(l - t) + 0.5w(l - t)^2\right)\right]}\\
&u^{*} = y_2[\rho l - p_2 + \ln(1 - \gamma_2 d_2 u_2)/\gamma_2 d_2]\\
&r_3^{*} = y_2[\rho l - p_2 + \ln(1 - \gamma_3 z_3 U_3)/\gamma_3 d_3]/[\lambda l + 0.5wl^2]\\
&m_1^{*} = y_1 \frac{\theta_1 \ln(1 - \gamma_1 z_1 U_1) + \gamma_1 d_1(\rho l + \rho t - p_1)}{\gamma_1 d_1(\beta_1 - \theta_1 - 1)\left[\lambda(l - t) + 0.5w(l - t)^2\right]}\\
&m_2^{*} = y_2 \frac{\theta_2}{\theta_2 + 1}\left[\rho l - p_2 - v(\lambda l + 0.5wl^2) + \ln(1 - \gamma_2 d_2 u_2)/\gamma_2 d_2\right]\\
&m_3^{*} = y_2 \frac{\theta_3}{\theta_3 - 1}(\lambda l + 0.5wl^2)\left[\rho l - p_2 - v + \ln(1 - \gamma_3 z_3 U_3)/\gamma_3 d_3\right]\\
&y_1 + y_2 = 1\\
&y_1 \leq z_1\\
&y_2 \leq z_2 + z_3
\end{aligned}$$

Again, z_i and y_i are binary variables. After optimizing the mixed integer mathematical model, the optimal decision variables in Manufacturer-Stackelberg are obtained.

3.2.2 Consumer-Stackelberg. Now we assume that the customer has more power than the manufacturer and the manufacturer has more power than the agent. Therefore, the customer selects the best strategy by dominating the manufacturer, and the manufacturer by dominating the agent. The related mixed integer programming for the agent as follower of the manufacturer is given by:

$$\begin{aligned}
&\max\Pi_A(r_1,u,r_3,m_1,m_2,m_3) = x_1d_1\left[(r_1-v)\left(\lambda(l-t)+0.5w(l-t)^2\right)-m_1\right] \\
&\qquad + x_2d_2\left[u-v(\lambda l+0.5wl^2)-m_2\right]+x_3d_3\left[(r_3-v)(\lambda l+0.5wl^2)-m_3\right] \\
&s.t.: \\
&\quad x_1\Pi_{M1} = d'\left[p_1+n-c-(c-s)(\lambda t+0.5wt^2)-h-\mu(b_1+b_2)\right] \\
&\quad (x_2+x_3)\Pi_{M2} = d''\left[p_2-c-h-\mu(b_1+b_2)\right] \\
&\quad q_1+q_2\leq 1 \\
&\quad x_1+x_2+x_3 = 1
\end{aligned} \tag{29}$$

x_i and q_i are binary variables.

According to the mixed integer programming similar to Manufacturer-Stackelberg, the optimal r_1*, u*, r_3*, m_1*, m_2* and m_3* are

$$r_1^* = \exp\left[\ln\left(x_1\Pi_{M1}/gp_1^{-\alpha_1}t^{-\alpha_1}n^{-\alpha_1}\begin{bmatrix}p_1+n-c- \\ (c-s)(\lambda t+0.5wt^2)-h-\mu(b_1+b_2)\end{bmatrix}\right)\Big/-\beta_1\right] \tag{30}$$

$$u^* = \exp\left[\ln\left(x_2\Pi_{M2}/gp_2^{-\alpha_2}[p_2-c-h-\mu(b_1+b_2)]\right)/-\beta_2\right] \tag{31}$$

$$r_3^* = \exp\left[\ln\left(x_3\Pi_{M3}/gp_2^{-\alpha_2}[p_2-c-h-\mu(b_1+b_2)]\right)/-\beta_3\right] \tag{32}$$

$$\begin{aligned}
m_1^* &= \frac{\theta_1\left[\lambda(l-t)+0.5w(l-t)^2\right]}{\theta_1+1} \\
&\times\left[\exp\left[\ln\left(x_1\Pi_{M1}/gp_1^{-\alpha_1}t^{-\alpha_1}n^{-\alpha_1}\begin{bmatrix}p_1+n-c-(c-s)(\lambda t+0.5wt^2) \\ -h-\mu(b_1+b_2)\end{bmatrix}\right)\Big/-\beta_1\right]-v\right]
\end{aligned} \tag{33}$$

$$m_2^* = \frac{\theta_2}{\theta_2+1}\begin{bmatrix}\exp\left[\ln\left(x_2\Pi_{M2}/gp_2^{-\alpha_2}[p_2-c-h-\mu(b_1+b_2)]\right)/-\beta_2\right] \\ -v(\lambda l+0.5wl^2)\end{bmatrix} \tag{34}$$

$$m_3^* = \frac{\theta_3}{\theta_3+1}(\lambda l+0.5wl^2)\left[\exp\left[\ln\left(x^3\Pi_{M3}/gp_2^{-\alpha_2}[p_2-c-h-\mu(b_1+b_2)]\right)/-\beta_3\right]-v\right] \tag{35}$$

As the customer is the leader of the manufacturer, based on the optimal variables, the satisfaction of the customer is also maximized:

$$\begin{aligned}
&\max U_C = \varepsilon_1 U_1 + \varepsilon_2 U_2 + \varepsilon_3 U_3 \\
&s.t.: \\
&\quad r_1^* = \varepsilon_1 \exp\left[\ln\left(x_1\Pi_{M1}/gp_1^{-\alpha_1}t^{-\alpha 1}n^{-\alpha_1}\left[p_1+n-c-(c-s)(\lambda t+0.5wt^2)-h-\mu(b_1+b_2)\right]\right)/-\beta_1\right] \\
&\quad u^* = \varepsilon_2 \exp\left[\ln\left(x_2\Pi_{M2}/gp_2^{-\alpha_2}[p_2-c-h-\mu(b_1+b_2)]\right)/-\beta_2\right] \\
&\quad r_3^* = \varepsilon_3 \exp\left[\ln\left(x_3\Pi_{M3}/gp_2^{-\alpha_2}[p_2-c-h-\mu(b_1+b_2)]\right)/-\beta_3\right] \\
&m_1^* = \frac{\varepsilon_1\theta_1\left[\lambda(l-t)+0.5w(l-t)^2\right]}{\theta_1+1} \\
&\quad \times\left[\exp\left[\ln\left(x_1\Pi_{M1}/gp_1^{-\alpha_1}t^{-\alpha_1}n^{-\alpha_1}\left[\begin{matrix}p_1+n-c-(c-s)(\lambda t+0.5wt^2)\\ -h-\mu(b_1+b_2)\end{matrix}\right]\right)/-\beta_1\right]-v\right] \\
&m_2^* = \frac{\varepsilon_2\theta_2}{\theta_2+1}\left[\exp\left[\mathrm{In}\left(x_2\Pi_{M2}/gp_2^{-\alpha_2}[p_2-c-h-\mu(b_1+b_2)]\right)/-\beta_2\right]-v(\lambda l+0.5wl^2)\right] \\
&m_3^* = \frac{\varepsilon_3\theta_3}{\theta_3+1}(\lambda l+0.5wl^2)\left[\exp\left[\mathrm{In}\left(x_3\Pi_{M3}/gp_2^{-\alpha_2}[p_2-c-h-\mu(b_1+b_2)]\right)/-\beta_3\right]-v\right] \\
&\varepsilon_1+\varepsilon_2+\varepsilon_3 = 1
\end{aligned} \tag{36}$$

ε_i are binary variables. When the above mathematical program is solved, the solution will determine the optimal decisions of the three echelons when the customer is the leader.

3.3 Cooperation

The previous three subsections discussed three non-cooperative games (one simultaneous-move game and two sequential-move games). In this part, we consider a cooperative game in which all the players, manufacturer, agent and customer, agree to cooperate and make joint decisions to maximize the profits of the whole system.

We consider that manufacturer, agent, and customer cooperate and act together as an integrated provider–service–customer. All the parties would participate in the cooperation only if their individual profits are higher than those of non-cooperative cases.

As we analyzed previously three scenarios – with warranty, without warranty but with maintenance cost and without warranty but with repair cost –, for each scenario $j = 1, 2, 3$, the profit function for the whole system is given by:

$$\begin{aligned}
\Pi_j^{co} &= \Pi_{Mj} + \Pi_{Aj} + \Pi_{Cj}, j = 1,2 \\
\Pi_j^{co} &= \Pi_{Mj-1} + \Pi_{Aj} + \Pi_{Cj}, j = 3
\end{aligned} \tag{37}$$

where Π_{Mj}, Π_{Aj} and Π_{cj} is the profit of manufacturer, agent and customer in scenario j, respectively.

The optimal decision variables can be obtained as follows:

$$p_1^{co} = \frac{\partial\Pi_1^{co}}{\partial p_1} \tag{38}$$

$$t^{co} = \frac{\partial \Pi_1^{co}}{\partial t} \tag{39}$$

$$p_2^{co} = \max\left\{\frac{\partial \Pi_2^{co}}{\partial p_2}, \frac{\partial \Pi_3^{co}}{\partial p_2}\right\} \tag{40}$$

$$r_1^{co} = \frac{\partial \Pi_1^{co}}{\partial r_1} \tag{41}$$

$$m_1^{co} = \frac{\partial \Pi_1^{co}}{\partial m_1} \tag{42}$$

$$u^{co} = \frac{\partial \Pi_2^{co}}{\partial u} \tag{43}$$

$$m_2^{co} = \frac{\partial \Pi_2^{co}}{\partial m_2} \tag{44}$$

$$r_3^{co} = \frac{\partial \Pi_3^{co}}{\partial r_3} \tag{45}$$

$$m_3^{co} = \frac{\partial \Pi_3^{co}}{\partial m_3} \tag{46}$$

All of these variables give the maximum profit for the whole system (as the benefit function is concave). However, none of the parties will be involved in cooperation unless the individual cooperative profits are higher than those of non-cooperative cases. They prefer to act jointly in cooperation provided their individual benefits are higher than non-cooperation.

Next, we discuss the feasibility of the cooperation. As argued by SeyedEsfahani *et al.* (2011), the cooperative solution is feasible if all the parties can obtain higher profit than other non-cooperative solutions, i.e. for all $j = 1, 2, 3$:

$$\Pi_{Mj}^{co} \geq \Pi_M^{\max} = \begin{Bmatrix} \max\{\Pi_{Mj}^S, \Pi_{Mj}^{MS}, \Pi_{Mj}^{CS}\}, j = 1, 2 \\ \max\{\Pi_{Mj-1}^S, \Pi_{Mj}^{MS}, \Pi_{Mj}^{CS}\}, j = 3 \end{Bmatrix} \tag{47}$$

$$\Pi_{Aj}^{co} \geq \Pi_A^{\max} = \max\left\{\Pi_{Aj}^S, \Pi_{Aj}^{MS}, \Pi_{Aj}^{CS}\right\} \tag{48}$$

$$\Pi_{Cj}^{co} \geq \Pi_C^{\max} = \max\left\{\Pi_{Cj}^S, \Pi_{Cj}^{MS}, \Pi_{Cj}^{CS}\right\} \tag{49}$$

where Π_{ij}^{co} is the value of the total profit of echelon i, $i = \{M, A, C\}$, under cooperation in secenario j, and Π_{ij}^S, Π_{ij}^{MS} Π_{ij}^{CS} is the total profit of echelon i in scenario j, $j = \{1, 2, 3\}$, under Static game, Manufacturer Stackelberg and Customer Stackelberg games, respectively.

Finally, the profit-sharing problem is formulated as the following bargaining problem:

$$\max_{j\in\{1,2,3\}}\left\{\begin{array}{l} \max_{\eta^i} \prod_{i\in\{M,A,C\}} \left(\pi_{ij}^{co} - \pi_i^{Max}\right)^{\eta^i} \\ s.t. \\ \pi_{ij}^{co} - \pi_i^{\max} \geq 0, \quad \forall j = 1, 2, 3 \\ \sum_{i\in\{M,A,C\}} \pi_{ij}^{co} = \pi_j^{co}, \quad \forall j = 1, 2, 3 \\ \sum_{i\in\{M,A,C\}} \eta^i = 1 \end{array}\right\} \tag{50}$$

where η_i is the bargaining power reached in the agreement for each echelon i under feasible $\left[\pi_{ij}^{co} - \pi_i^{\max} \geq 0, \quad \forall j = 1, 2, 3\right]$ cooperation $\left[\sum_{i\in\{M,A,C\}} \pi_{ij}^{co} = \pi_j^{co}, \quad \forall j = 1, 2, 3\right]$.

Then, for each echelon $i = \{M, A, C\}$, the optimal extra-profit of cooperation will be:

$$p_i^{co} = \eta_i^* \max_{j\in\{1,2,3\}}\left\{\prod_{i\in\{M,A,C\}} \left(\pi_{ij}^{co} - \pi_i^{\max}\right)^{\eta_i^*}\right\},$$

with η_i^* being the optimal bargaining power reached in (49).

4. Numerical examples

To illustrate the supply games developed, we consider several numerical examples. The input data are adapted from Esmaeili *et al.* (2014) as follows: $\lambda = 0.1$, $w = 0.3$, $l = 6$, $v = 1{,}000$, $u = 800$, $\delta = 4{,}500$, $s = 500$, $g = 10$, $\alpha_1 = 1.2$, $\alpha_2 = 1.5$, $\beta_1 = 1.2$, $\beta_2 = 1.5$, $\beta_3 = 1.2$, $\theta_1 = \theta_2 = \theta_3 = 0.15$, $b_1 = b_2 = 50$, $h = 300$, $\gamma = 0.2$, $\mu = 500$ and $\rho = 15{,}000$.

4.1 Example 1 (static game)

The first example is given by $N = \{M,A,C\}$, $S_M = \{M_1, M_2\}$, $S_A = \{A_1, A_2, A_3\}$, $S_C = \{C_1, C_2, C_3\}$, and $\prod(M) = \{1.57, 0.73\}$, $\prod(A) = \{9.92, 0.57, 7.64\}$, $\prod(C) = \{5, 4.98, 5\}$.

Table III illustrates the interaction among the players and the Nash equilibrium (A_1, C_1, M_1) con payoff vector (1.57, 9.92, 5).

We can conclude that the best strategy is when both the manufacturer and the agent choose option 1, i.e. the warranty service is offered. If the customer accepts the warranty

Table III. Profits of the manufacturer and the agent, and the utility function of customer

		C_1	C_2	C_3
A_1	M_1	(1.57, 9.92, 5)	(1.57, 9.92,4.98)	(1.57, 9.92,5)
	M_2	(0.73, 9.92, 5)	(0.73, 9.92, 4.98)	(1.4, 9.92,5)
A_2	M_1	(1.57, 0.57, 5)	(1.57, 0.57, 4.98)	(1.57, 0.57,5)
	M_2	(0.73, 0.57, 5)	(0.73, 0.57, 4.98)	(1.4, 0.57,5)
A_3	M_1	(1.57, 7.64, 5)	(1.57, 7.64, 4.98)	(1.57, 7.64,5)
	M_2	(0.73, 7.64, 5)	(0.73, 7.64, 4.98)	(1.4, 7.64,5)

service, he reaches his highest value of satisfaction, while the manufacturer and the agent obtain the maximal profit.

4.2 Example 2 (Manufacturer-Stackelberg)

The second example considers the manufacturer as the leader, followed by the agent and then the customer. The following optimal values are obtained for this case:

$$t^* = 16.63 \quad n^* = 10 \quad p_1^* = 9.1 \quad r_1^* = 2.4 \quad d_1^* = 1 \quad m_1^* = 4.77 \quad \Pi_{M1} = 1.57 \quad \Pi_{A1} = 9.92$$

$$p_2^* = 3.3 \quad u^* = 2.57 \quad d_2^* = 1 \quad m_2^* = 0.26 \quad \Pi_{M2} = 0.73 \quad \Pi_{A2} = 0.57 \quad U_{c2} = 4.98 \quad U_{c1} = 5$$

$$p_2^* = 3.3 \quad r_3^* = 2.4 \quad d_3^* = 1 \quad m_3^* = 1.8 \quad \Pi_{M2} = 1.4 \quad \Pi_{A3} = 7.64 \quad U_{c3} = 5$$

It is inferred here that the manufacturer chooses to offer the warranty service and the highest profit value is obtained. Moreover, the agent as follower of the manufacturer and leader of the customer prefers option 1. Then, the customer, who may choose option 1 or 3, is forced by his leader to select the service warranty.

4.3 Example 3 (Consumer-Stackelberg)

This example considers the customer as the leader, followed by the manufacturer and then the agent (Esmaeili *et al.*, 2014). The optimal values are presented as follows:

$$t^* = 16.63 \quad n^* = 10 \quad p_1^* = 9.1 \quad r_1^* = 2.4 \quad d_1^* = 1 \quad m_1^* = 4.77 \quad \Pi_{M1} = 1.57 \quad \Pi_{A1} = 9.92$$

$$p_2^* = 3.3 \quad u^* = 2.57 \quad d_2^* = 1 \quad m_2^* = 0.26 \quad \Pi_{M2} = 0.73 \quad \Pi_{A2} = 0.57 \quad U_{c2} = 4.98 \quad U_{c1} = 5$$

$$p_2^* = 3.3 \quad r_3^* = 2.4 \quad d_3^* = 1 \quad m_3^* = 1.8 \quad \Pi_{M2} = 1.4 \quad \Pi_{A3} = 7.64 \quad U_{c3} = 5$$

The above results show that the customer can choose between the warranty services and repair the failures that occur during the lifetime of the product at a fixed cost per failure (the satisfaction level in both cases is 5). Nevertheless, if the customer chooses warranty, his followers also get a better profit. Consequently, the best strategy here is that all of the echelons in the supply chain prefer the first option, i.e. providing the warranty service.

4.4 Example 4 (bargaining game)

In the last example, the cooperative model is applied. Some optimal values of the objective function are illustrated in Table IV according to different values of the bargaining power.

	η_M	η_A	η_C	Bargaining problem objective function value		
				With warranty	With repair cost	With maintenance cost
1	0.8	0.1	0.1	0.0071	0.1263	0.0923
2	0.7	0.2	0.1	0.0104	0.1100	0.1228
3	0.6	0.3	0.1	0.0153	0.0958	0.1632
4	0.5	0.4	0.1	0.0224	0.0835	0.2170
5	0.4	0.5	0.1	0.0328	0.0727	0.2884

Table IV. The objective function under various values of the bargaining power

Table IV shows the sensitivity analysis of the bargaining powers in the various scenarios. The results here are slightly different. The agent has half of the bargaining power, while the manufacturer has more power (0.4) than the consumer (0.1). The best strategy is that all of the echelons prefer no warranty service with maintained cost, and the optimal extra-profits are 0.1085, 0.0868 and 0.0217 for manufacturer, agent and consumer, respectively.

5. Sensitivity analysis

To complete our study of the three-level supply chain with marketing and warranty services, we present a sensitivity analysis of the selection of options under the different values of some parameters of the model.

The four experiments are designed to investigate the effects of risk parameter, repair cost, lifetime, amplitude constant factor and rate of ageing as follows: the parameter (1) is decreased by −50 per cent, parameter (2) decreased by −25 per cent, parameter (3) increased by +25 per cent and parameter (4) increased by +50 per cent. The produced values by sensitivity analysis are reported in Tables V-VII and Figures 1-10.

The effect of the risk parameter on the customer's utility is shown in Figure 1. Notice that if the risk parameter increases, the risk aversion of the customer decreases, and, after a certain value, there is no difference between the three options for the customer.

Figures 2 and 3 show the effect of the repair cost on the agent's profit and the customer's utility under the different options, respectively. As Esmaeili *et al.* (2014), here it is inferred

Table V. The effect of the risk parameters, repair cost, lifetime on the objective functions

			γ			V			v			l	
		u_{c1}	u_{c2}	u_{c3}	Π_{A1}	Π_{A2}	Π_{A3}	u_{c1}	u_{c2}	u_{c3}	Π_{A1}	Π_{A2}	Π_{A3}
1	−50%	6.407	9.376	9.900	10.267	0.721	1.318	4.994	5.000	5.000	17.865	1.888	1.731
2	−25%	5.231	6.563	6.660	10.060	0.625	7.752	4.860	4.999	5.000	13.563	0.684	4.094
3	+25%	3.690	3.996	4.000	9.807	0.523	7.556	3.415	4.915	4.995	6.901	0.487	12.473
4	+50%	3.179	3.333	3.333	9.718	0.491	7.488	2.144	4.788	4.978	4.486	0.431	18.674

Table VI. The effect of the amplitude constant factor on the objective functions

					G					
		Π_{M1}	Π_{M2}	Π_{M3}	Π_{A1}	Π_{A2}	Π_{A3}	u_{c1}	u_{c2}	u_{c3}
1	−50%	0.786	0.363	0.701	4.958	0.283	3.821	3.203	4.688	4.950
2	−25%	1.180	0.544	1.051	7.438	0.424	5.731	3.923	4.922	4.995
3	+25%	1.966	0.907	1.752	12.396	0.707	9.551	5.000	4.995	4.613
4	+50%	2.359	1.089	2.102	14.875	0.848	11.462	4.768	4.999	5.000

Table VII. The effect of the rate of ageing on the objective functions

					W					
		Π_{M1}	Π_{M2}	Π_{M3}	Π_{A1}	Π_{A2}	Π_{A3}	u_{c1}	u_{c2}	u_{c3}
1	−50%	0.786	0.363	0.701	4.958	0.283	3.821	3.203	4.688	4.950
2	−25%	1.180	0.544	1.051	7.438	0.424	5.731	3.923	4.922	4.995
3	+25%	1.966	0.907	1.752	12.396	0.707	9.551	5.000	4.995	4.613
4	+50%	2.359	1.089	2.102	14.875	0.848	11.462	4.768	4.999	5.000

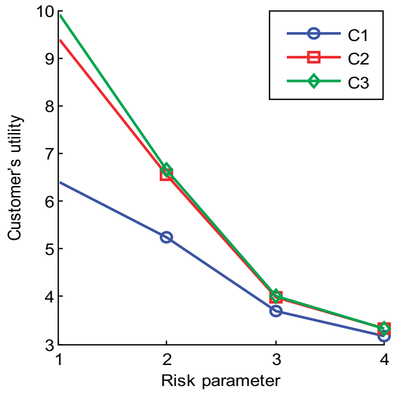

Figure 1.
The effect of the risk parameter on customer's utility

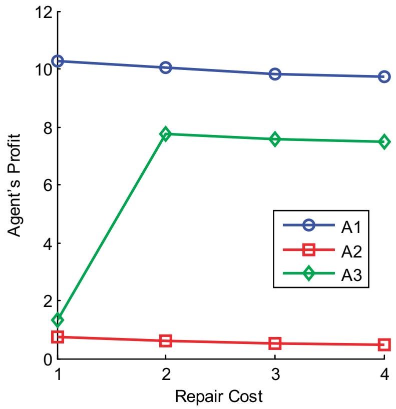

Figure 2.
The effect of the repair cost on the agent's profit

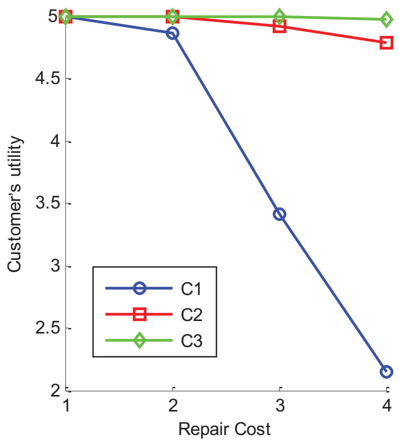

Figure 3.
The effect of the repair cost on the customer's utility

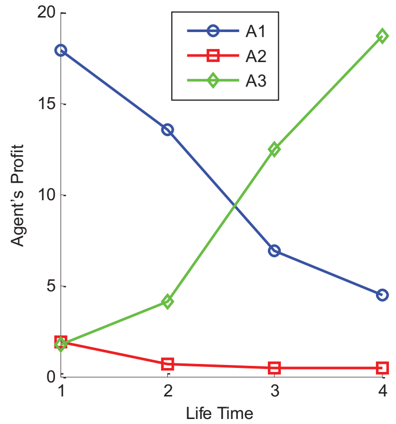

Figure 4.
The effect of the lifetime on the agent's profit

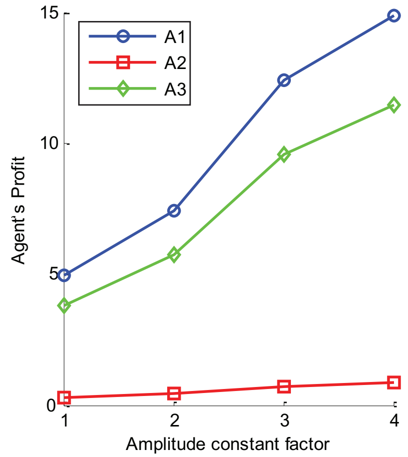

Figure 5.
The effect of the amplitude constant factor on the agent's profit

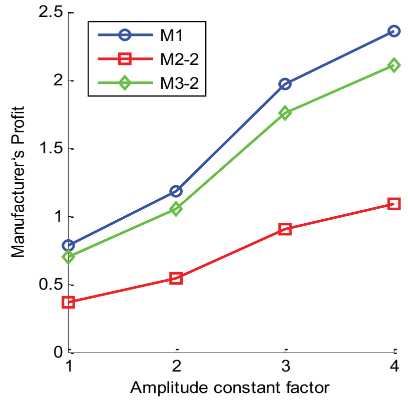

Figure 6.
The effect of the amplitude constant factor on the manufacture's profit

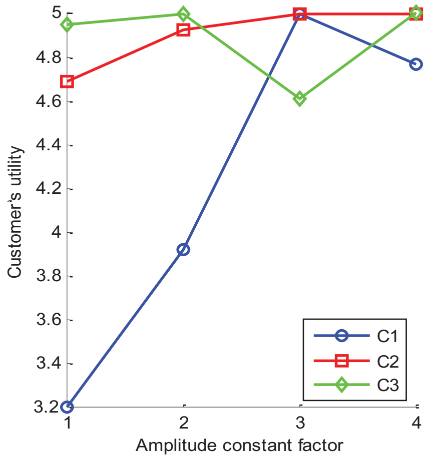

Figure 7.
The effect of the amplitude constant factor on the customer's utility

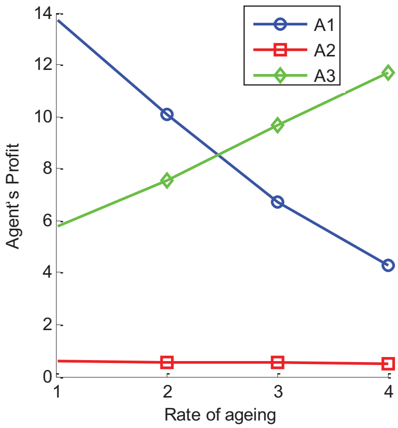

Figure 8.
The effect of the rate of ageing on the agent's profit

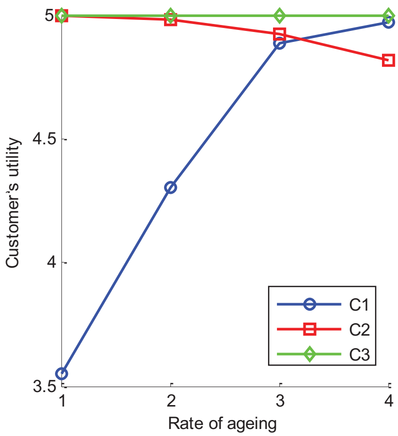

Figure 9.
The effect of the rate of ageing on the customer's utility

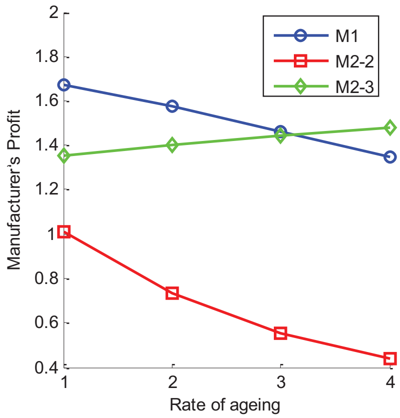

Figure 10.
The effect of the rate of ageing on the manufacture's profit

that first option always dominated two other options. It is true that by increasing the repair cost, the utility decreases, as Figure 3 shows.

As Figure 4 illustrates the effect of the lifetime on the agent's profit, before and after 2.5 years; A1 and A3 dominate the other options. A2 is approximately independent on the variation of lifetime because the related cost is paid at first.

Figures 5-7 display the effect of the amplitude constant factor on the agent's profit, the manufacturer's profit and the customer's utility, respectively. Figures 5 and 6 are quite similar, and by increasing the amplitude, constant factor also increases.

Figures 8-10 show the effect of the rate of ageing on the agent's profit, the customer's utility and the manufacturers profit, repectivly. As it can be seen, the objective functions values are sensitive to the rate of ageing and are varied by the variation in the rate of agening.

6. Case study and managerial implications

If the decisions of a supply chain are made accurately and properly, the managerial abilities are promoted and consequently the most critical and major problems can be resolved. Some activities that can be used to achieve this goal and can help to improve the overall effectiveness of the whole supply chain and maximize the total profit are warranty, marketing and pricing decisions. In this research, it has been attempted to consider these important decisions simultaneously and help the managers. This study is an advisable option and can help managers and stakeholders to improve the performance of the supply chain. Here, to verify this claim, the suggested model is analyzed through conducting a case study in automotive industry. It is demonstrated that the proposed model can be implemented successfully in practice and it would be the beneficial guide for the managers. Iran Khodro, branded as IKCO, is an Iranian automaker founded in 1962. It annually produces about 688,000 passenger cars. IKCO manufactures vehicles, including Samand, Peugeot and Renault cars, and trucks, minibuses and buses. IKCO Spare Parts and After-Sale Services Co. (ISACO) was founded in 1977. ISACO is an agent providing after-sale services and supplying spare parts for IKCO products throughout the country[1]. The customers face three options with warranty (giving a Golden card), without warranty but with maintenance cost and without warranty but with repair cost. So the studied case can be formulated and optimized under the proposed models. The optimal results are reported as follows.

First the interaction among the players and the Nash equilibrium (A_1, C_1, M_1) are illustrated in Table VIII. The best option can be offering of warranty service as it is shown in Table VIII. The maximum satisfaction of the customer as well as the maximum profit of both the manufacturer and the agent can be obtained by offering of warranty service.

Now the optimal values of the decision variables obtained by Stackelberg models are presented as follows:

$$t^* = 13.33 \quad n^* = 8 \quad p_1^* = 71.8 \quad r_1^* = 6 \quad d_1^* = 1 \quad m_1^* = 5.5 \quad \Pi_{M1} = 1.26 \quad \Pi_{A1} = 14.23$$

$$p_2^* = 22.5 \quad u^* = 6.64 \quad d_2^* = 1 \quad m_2^* = 0.66 \quad \Pi_{M2} = 0.15 \quad \Pi_{A2} = 0.14 \quad U_{c2} = 2.76 \quad U_{c1} = 6.16$$

$$p_2^* = 22.5 \quad r_3^* = 6 \quad d_3^* = 1 \quad m_3^* = 4.65 \quad \Pi_{M2} = 1.09 \quad \Pi_{A3} = 11.83 \quad U_{c3} = 4.56$$

It is found that the results obtained from the first strategy outperform of others. So providing of warranty service is preferred in these models.

Finally, the various scenarios are studied with bargaining game. The bargaining powers of the agent, the manufacturer and the consumer are considered 0.5, 0.4 and 0.1, respectively. The extra-profits are 0.911, 0.306 and 1.063 for manufacturer, agent and consumer, respectively.

The main suggestion to the managers is consideration of a third party to provide warranty service. As is shown above, this promotes the profit of the supply chain and customer satisfaction. The best strategy for the studied case is offering the Golden card to customer to use warranty services by IKCO.

7. Conclusions and further research

A game-theoretic approach (non-cooperative and cooperative) for three-echelon supply chains with marketing and warranty services is presented in this paper. The non-cooperative approaches are both static (Nash equilibrium) and dynamic (Stackelberg) models. The cooperative approach related to bargaining models (Nash bargaining games).

To the best of the authors' knowledge, a research investigating the warranty, marketing and pricing issues simultaneously has not been addressed in the literature yet. In addition, the main contributions of this paper that differentiate it from the existing papers are regarding inventory, lost sale and lost goodwill, which are crucial in the comparison environment. Considering the risk parameter brings the proposed model closer to the real world. So the proposed model is more general, in line with the real world, and it overcomes many of the presented research gaps and the suggestions of the previous researchers. Several numerical examples and a case study are presented to illustrate the applications of

		C_1	C_2	C_3
A_1	M_1	(1.26, 14.23, 6.16)	(1.26, 14.23, 2.76)	(1.26, 14.23, 4.56)
	M_2	(0.15, 14.23, 6.16)	(0.15, 14.23, 2.76)	(1.09, 14.23, 4.56)
A_2	M_1	(1.26, 0.14, 6.16)	(1.26, 0.14, 2.76)	(1.26, 0.14, 4.56)
	M_2	(0.15, 0.14, 6.16)	(0.15, 0.14, 2.76)	(1.09, 0.14, 4.56)
A_3	M_1	(1.26, 11.83, 6.16)	(1.26, 11.83, 2.76)	(1.26, 11.83, 4.56)
	M_2	(0.15, 11.83, 6.16)	(0.15, 11.83, 2.76)	(1.09, 11.83, 4.56)

Table VIII. Profits of the manufacturer and the agent, and the utility function of customer in the studied case the game theoretical models presented. Finally a sensitivity analysis of the parameters, including risk parameter, repair cost, lifetime, amplitude constant factor and rate of ageing, which were useful for analyzing results, is developed. Through a case study, it is demonstrated that consideration of a third party to provide warranty service could help to supply chain managers because, as it was shown, this can promote the profit of the supply chain and customer satisfaction.

One of the limitations of the proposed models is considering the parameters as certain. Considering different types of uncertainty for the parameters can be another interesting topic for future research. Moreover, imposing reverse logistics and closed-loop supply chain can be other significant issues. Further research may try to accomplish a model considering other challenges such as reliability, besides warranty. The proposed models may be improved with a large-scale database. Moreover we suggest researchers to incorporate the concept of this work to the research studies of Taleizadeh and Pentico (2014), Taleizadeh (2014) and Taleizadeh *et al.* (2015).

Note

1. https://en.wikipedia.org/wiki/Iran_Khodro

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Appendix 1

AP1. To prove that the profit function shown in equation (1) is concave, we apply the following well-known result.

Consider a function F with two variables (x_1 and x_2) and $H = \begin{bmatrix} \frac{\partial^2 F}{\partial x_1^2} & \frac{\partial^2 F}{\partial x_1 \partial x_2} \\ \frac{\partial^2 F}{\partial x_2 \partial x_1} & \frac{\partial^2 F}{\partial x_2^2} \end{bmatrix}$, F is a concave function if $X.H.X^T < 0$.

Then we have to show that the following condition is satisfied:

$$(p_1,\, t) = \begin{bmatrix} \frac{\partial^2 \Pi_{M1}}{\partial p_1^2} & \frac{\partial^2 \Pi_{M1}}{\partial p_1\, \partial t} \\ \frac{\partial^2 \Pi_{M1}}{\partial t\, \partial p_1} & \frac{\partial^2 \Pi_{M1}}{\partial t^2} \end{bmatrix} \begin{pmatrix} p_1 \\ t \end{pmatrix} < 0 \tag{A1}$$

After simplification of (A.1), we have:

$$\begin{aligned} &-6p_1\alpha_1 g - 2t(2p_1\alpha_1 g + 1)\left[\delta - (\lambda + wt)(c - s)\right] \\ &-3\alpha_1 g - (3\alpha_1 + 1)\left[c + h - p_1 + \mu(b_1 + b_2) - \delta t + (0.5wt^2 + \lambda t)(c - s)\right] < 0 \end{aligned} \tag{A2}$$

Consequently profit function (1) is concave if (A2) holds.

AP2. $\Pi_{M2}(P_2)$ is concave if and only if $\delta^2\Pi_{M2}(P_2)/\delta p_2{}^2 \le 0$ for all $p_2 \ge 0$:

$$\delta^2\ \Pi_{M2}(P_2)/\delta p_2^2 \le 0 \leftrightarrow$$
$$-\alpha_2 g\left[2+\ (\alpha_2+1)\,c+h+p_2+\mu\,(b_1+b_2)\right]<0 \tag{A3}$$

AP3. According to (Esmaeili *et al.*, 2009) the concavity of function shown in equation (6) can be shown as follows. The function F is strictly pseudoconcave function if -F is a strictly pseudoconvex function. $F(x_1, x_2)$ is a pseudoconvex function if either $\nabla F(x_1)(x_2 - x_1) \ge 0$ then $F(x_1) > F(x_2)$ or if $F(x_1) \le F(x_2)$ then $\nabla F(x_1)(x_2 - x_1) < 0$. Now we try show that $\Pi_{A1}(r_1^1, m_1)$ is strictly pseudoconcave with respect to r_1 for a fixed m_1. $\Pi_{A1}(r_1^2, m_1) \le \Pi_{A1}(r_1^1, m_1)$ can be rewritten as follows:

$$d_1^2\left[\left(v-r_1^2\right)\left(\lambda\,(l-t)+0.5w(l-t)^2\right)+m_1\right] \le d_1{}^1\left[\left(v-r_1{}^1\right)\left(\lambda\,(l-t)+0.5w(l-t)^2\right)+m_1\right] \tag{A4}$$

If $\nabla\ \Pi_{A1}(r_1{}^1, m_1)(r_1{}^2 - r_1{}^1) < 0$, $\Pi_{A1}(r_1, m_1)$ is strictly pseudoconcave. In other words:

$$d_1{}^1 r_1{}^2\left(\lambda\,(l-t)+0.5w\,(l-t)^2\right)(\beta_1-1)-d_1{}^1\beta_1\left[v\left(\lambda\,(l-t)+0.5w(l-t)^2\right)+m_1\right]<0 \tag{A5}$$

Condition (A5) will be true if the following two inequalities hold:

$$r_1{}^2 < r_1{}^1 \tag{A6}$$

$$(v-r_1{}^i)\left(\lambda\,(l-t)+0.5w\ (l-t)^2\right)+m_1>0 \quad i=1,2 \tag{A7}$$

Suppose that (A6) is not true, i.e. $r_1{}^2 < r_1{}^1$, then $d_1{}^i(r_1{}^i, m_1) = gp_1^{-\alpha_1}(r_1{}^i)^{-\beta 1} m_1{}^{\theta_1}$, hence $d_1{}^1 \le d_1{}^2$ and we also have:

$$d_1{}^1\left[v-r_1{}^2\right)\left(\lambda\,(l-t)+0.5w(l-t)^2+m_1\right]$$
$$\le d_1{}^2\left[(v-r_1{}^1)\left(\lambda\,(l-t)+0.5w\ (l-t)^2\right)+m_1\right] \tag{A8}$$

Because (A8) contradicts (A4), (A6) is proved. Similarly suppose that (A8) is not true, then $(v-r_1{}^i)\left(\lambda\,(l-t)+0.5w\ (l-t)^2\right)+m_1 \le 0 \quad i=1,2$ and $r_1{}^2 > r_1{}^1$ leads to:

$$(v-r_1{}^1)\left(\lambda\,(l-t)+0.5w(l-t)^2\right)+m_1 \le (v-r_1{}^2)\left(\lambda\,(l-t)+0.5w\ (l-t)^2\right)+m_1 \tag{A9}$$

Because $d_1^2 < d_1^1$, consequently we have:

$$d_1^1[v-1)r_1^1\left(\lambda\,(l-t)+0.5w\,(l-t)^2+m_1\right) \le d_1^2\left[\left(v-r_1^1\right)\left(\lambda\,(l-t)+0.5w\,(l-t)^2\right)+m_1\right], \tag{A10}$$

which contradicts (A4), thus (A6) holds.

Finally, (A7) implies $(v - r_1{}^1)\left(\lambda\,(l-t) + 0.5w\ (l-t)^2\right) + m_1 > 0$ and it is can be rewritten as follows:

$$v\left(\lambda\,(l-t) + 0.5w\ (l-t)^2\right) + m_1 > r_1{}^1\left(\lambda\,(l-t) + 0.5w\ (l-t)^2\right) \tag{A11}$$

According to $0 < \beta_1 - 1 < \beta_1$ and (A11) we have:

$$(\beta_1 - 1)r_1^1\left(\lambda\,(l-t) + 0.5w\ (l-t)^2\right) < \beta_1\left[v\ \left(\lambda\,(l-t) + 0.5w\ (l-t)^2\right) + m_1\right] \tag{A12}$$

Since $d_1^1 \geq 0$, the following inequality holds:

$$d_1^1[\beta_1 - 1)r_1^1\left(\lambda\,(l-t) + 0.5w\,(l-t)^2\right) < d_1^1\beta_1\left[v\ \left(\lambda\,(l-t) + 0.5w\ (l-t)^2\right) + m_1\right] \tag{A13}$$

As a result (A5) is proved and then $\Pi_{M2}(p_2)$ is a strictly pseudoconvex function, hence $\Pi_{M2}(p_2)$ is a strictly pseudoconcave function.

AP4. To prove the concavity of (9) we use a similar argument to that used in AP.1. That is, (9) is concave if $\theta_2 + 1 < \alpha_2 < \beta_2$ and:

$$\frac{(\theta_2 - \beta_2)\left[2(\beta_2 - \theta_2)(\alpha_2 - \theta_2) + \beta_2 - \alpha_2 + 1\right]}{\alpha_2 - \theta_2 - 1} < 0 \tag{A14}$$

AP5. A similar argument to *AP1* is used to prove the concavity of (12). Then if

$$\frac{(\theta_2 - \beta_2)\left[(\beta_3 - \theta_3)(\beta_3 - \theta_3 + 2) - 1\right]}{\beta_3 - \theta_3 - 1} < 0 \tag{A15}$$

And $\theta_3 + 1 < \beta_3$ and $\theta_2 < \beta_2$ equation (12) is concave.

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